Conference on Algebraic Geometry "Sputnik-22"

June 27–July 1, 2022, Moscow, Russia

Steklov Mathematical Institute of RAS

Valery Alexeev. Compact moduli spaces of $K3$ surfaces

I will discuss several recent results, most of them joint with Philip Engel, that relate functorial, geometrically meaningful compactifications of moduli spaces of $K3$ surfaces with toroidal and semi-toroidal compactifications.

Alexey Bondal. Two derived categories of a generic complex torus

We will show that the derived category of $\mathcal{O}$-modules with coherent cohomology and the derived category of coherent sheaves are not equivalent for a generic compact complex-analytic torus of dimension > 2. We will compare the algebraic structure of the two categories.

Alexander Efimov. K-theory of inverse limits

We will recall the recent notion of a Mittag–Leffler inverse sequence of DG categories, and sketch a proof that K-theory specturm of a suitably defined limit is identified with the inverse limit of K-theory spectra. This in particular applies to the category of nuclear modules on a formal scheme.

Bernhard Keller. On the Donovan–Wemyss conjecture, after Wemyss, Jasso–Muro. . .

We will give an introduction to Donovan–Wemyss conjecture stating that a compound Du Val singularity is determined by the derived equivalence class of the contraction algebra associated with any crepant resolution (Donovan–Wemyss 2013). More and more evidence for the conjecture has been accumulated in work by Donovan–Wemyss, Wemyss, Toda, Hua–Toda, Hua, August, Hua–K and others. We will conclude by explaining how the conjecture follows by combining the work of August and Hua–K with the Auslander–Iyama correspondence for triangulated categories, a recent theorem due to Gustavo Jasso and Fernando Muro.

Victor Kulikov. On Chisini theorems for covers of the projective plane

In the talk, it will be discussed the question when finite covers of the projective plane are uniquely determined by the local data on the behavior of covers over the points of their branch curves.

Valery Lunts. Some conjectures about LLV Lie algebra, group of autoequivalences of the derived category and monodromy

I will discuss some recent work on the conjectural relation of the group of autoequivalences of the derived category and the LLV Lie algebra for (weak) CY varieties. Another conjecture (essentially due to Kontsevich) describes the relation of the above group to the monodromy of the mirror symmetric family.

Amnon Neeman. Vanishing K-theory and bounded t-structures

We will begin with a quick reminder of algebraic K-theory, and a gentle review of some classical results. The talk will then focus on a striking 2019 article by Antieau, Gepner and Heller — it turns out that there are K-theoretic obstructions to the existence of bounded t-structures.

The result suggests many questions. A few have already been answered, but many remain open. We will concentrate on the many possible directions for future research.

Martin Olsson. Derived equivalences over base schemes and support of complexes

Let X and Y be smooth projective varieties over a field k, equipped with morphisms $f\colon X \to T$ and $g\colon Y \to T$ to another scheme T. I will discuss conditions on a derived equivalence given by a kernel $P \in D(X \times Y)$ ensuring that P is in the image of an object in $D(X \times_T Y)$ giving derived equivalences among the fibers. This is joint work with Max Lieblich.

John Ottem. Tropical degenerations and stable rationality

I will explain how tropical degenerations and birational specialization techniques can be used in rationality problems. In particular, I will apply these techniques to study quartic fivefolds and complete intersections of a quadric and a cubic in $\mathbb{P}^6$. This is joint work with Johannes Nicaise.

Ivan Panin. On Suslin's exact sequence in mixed characteristic

The Suslin exact sequence relates a part of the 3rd étale cohomology group to the corresponding torsion subgroup in codimension two cycles. We prove a mixed characteristic version of this result.

Alex Perry. Derived categories of Fano threefolds

Kuznetsov conjectured the existence of a correspondence between different types of Fano threefolds which identifies a distinguished semiorthogonal component of the derived category on each side. I will explain joint work with Arend Bayer which resolves the last outstanding case of this conjecture. The

proof relies on the study of certain equivariant categories and their Hodge theory, and gives as a byproduct a "birational categorical Torelli theorem" for a class of Fano threefolds.

Yuri Prokhorov. Singular Del Pezzo varieties

A del Pezzo variety X is a Fano variety whose anticanonical class has the form $-K_X = (n-1)A$, where A is an ample line bundle and n is the dimension of X. This is a higher dimensional analog of the notion of del Pezzo surfaces. I am going to discuss biregular and birational classifications of del Pezzo varieties admitting terminal singularities.

The talk is based on a joint work with Alexander Kuznetsov.

Giulia Sacca. Moduli spaces as Irreducible Symplectic Varieties

Recent developments by Druel, Greb–Guenancia–Kebekus, Horing–Peternell have led to the formulation of a decomposition theorem for singular (klt) projective varieties with numerical trivial canonical class. Irreducible symplectic varieties are one the building blocks provided by this theorem, and the singular analogue of irreducible hyper-Kahler manifolds.

In this talk I will show that moduli spaces of Bridgeland stable objects on the Kuznetsov component of a cubic fourfold with respect to a generic stability condition are always projective irreducible symplectic varieties. This builds on the recent work of Bayer–Lahoz–Macri–Neuer–Perry–Stellari, which, ending a long series of results by several authors, proved the analogue statement in the smooth case.

Evgeny Shinder. Derived Jacobians of elliptic $K3$ surfaces

We address the question of Hassett–Tschinkel on whether all Fourier–Mukai partners of an elliptic $K3$ surface are among their relative Jacobians. The answer is positive in the coprime rank two case, but negative in general. The proofs involve Brauer groups, transcendental lattices of $K3$ surfaces, and the counting formula for Fourier–Mukai partners. On the conceptual level the structures we study can be thought of as derived Jacobians of a $K3$. This is joint work in progress with Reinder Meinsma.

Constantin Shramov. Conic bundles

Consider a conic bundle over a smooth incomplete curve C, i.e. a smooth surface S with a proper surjective morphism to C such that the push-forward of the structure sheaf of S coincides with the structure sheaf of C, and the anticanonical class of S is ample over C. I will tell about a necessary and sufficient condition for the existence of an extension of this conic bundle to

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the completion of C. The talk is based on a joint work in progress with V. Vologodsky.

Yukinobu Toda. Categorical and K-theoretic Donaldson–Thomas theory of $\mathbb{C}^3$

The generating series of Donaldson–Thomas invariants associated with Hilbert schemes of points on the three-dimensional affine space is known to form a MacMahon function, whose coefficients are numbers of plane partitions. In this talk, I will give a categorical and K-theoretic analogue of the above formula. I will consider the triangulated categories of matrix factorizations of super-potentials whose critical loci are Hilbert schemes of points, and show the existence of their semi-orthogonal decompositions which are regarded as categorification of the above MacMahon formula. In fact there exist explicitly constructed objects in each semi-orthogonal summand, whose total number is the number of plane partitions, which generate the torus localized K-group of the above category of matrix factorizations. This is a joint work in progress with Tudor Padurariu.

Yuri Tschinkel. Linear and stably linear actions on rational varieties

I will discuss new invariants and constructions in equivariant birational geometry (joint work with B. Hassett and A. Kresch).

Claire Voisin. A topological characterization of hyper-Kähler fourfolds of $\mathrm{Hilb}^2(K3)$ type

There are two known deformation types of hyper-Kähler (HK) fourfolds, namely $\mathrm{Hilb}^2(K3)$ (Beauville, Fujiki) and the generalized Kummer variety $K_2(A)$ (Beauville). It is however still unknown whether there are other topological types or deformation types of HK fourfolds. Some strong topological restrictions on HK fourfolds are known by work of Beauville, S. Salamon, Verbitsky and Guan. In this talk I will sketch the proof of the following result

Thm. A hyper-Kähler fourfold X is a deformation of $\mathrm{Hilb}^2(K3)$ if and only if it has two integral degree 2 cohomology classes l and m satisfying the conditions $l^4 = 0$, $m^4 = 0$, $l^2m^2 = 2$. In particular, a HK fourfold which is homeomorphic to $\mathrm{Hilb}^2(K3)$ is a deformation of $\mathrm{Hilb}^2(K3)$.

This is joint work with Debarre, Huybrechts and Macrì.

Chenyang Xu. K-stability theory of Fano varieties

K-stability of Fano varieties was initiated as a central topic in complex geometry, for its relation with the Kahler–Einstein problem. It turns out that

the machinery of higher dimensional geometry, centered around the minimal model program, provides a fundamental tool to study it, and therefore makes it an active algebraic topic. This meeting of two well-studied fields has made a number of major conjectures solved, including the Yau–Tian–Donaldson Conjecture, and the construction of a moduli space for Fano varieties. In the talk, we will survey the recent progress.