## From the B-Toda to the BKP hierarchy

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In this talk we will show that all  $\tau$ -functions of BKP hierarchy can be written as Pfaffians of skew-symmetric matrices.  $\tau$ -functions of BKP hierarchy are parameterized by points in the universal orthogonal Grassmannian manifold (UOGM). The UOGM is a disjoint union of Schubert cells, we classify and give explicit parameterization for points in each Schubert cell by constructing a frame for UOGM in the sense of Sato.  $\tau$ -functions are then expressed in terms of these frames and Schur-Q functions. For concreteness we give a comprehensive study for the  $\tau$ -functions of *B*-Toda which can be viewed as a finite version of the BKP hierarchy. Along the way we also give a constructive description for complex pure spinors du E. Cartan. As an application of our construction, we reprove a theorem due to A. Alexandrov which states that  $\tau$ -functions of KdV satisfy BKP up to rescaling of the time parameters by 2. We prove this by showing that the KdV hierarchy can be viewed as 4-reduction of the BKP hierarchy. This interpretation gives complete characterization for the KdV orbits inside the BKP hierarchy. Other than a few facts from representation theory, the main tools we use to show the above results, however, are surprisingly simple linear algebra. This talk is based on a recent preprint arXiv:2210.03307.

Yuancheng Xie is currently a postdoc at BICMR, and he obtained his Ph.D. degree from The Ohio State University in 2021 under the guidance of Yuji Kodama. His current interests lie in integrable systems and related algebras and geometries.